SCORE: ____ / 30 POINTS 4+

1. NO CALCULATORS OR NOTES ALLOWED
2. SHOW PROPER CALCULUS-LEVEL WORK
3. SIMPLIFY ALL ANSWERS

Find $\frac{d}{dx}\frac{\tanh^{-1}(x^3)}{x^2}$. Simplify your final answer as a single fraction.

SCORE: 2 / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$\left(\frac{1}{1-x^6}\right)(x^2) - (2x)(\tanh^{-1}(x^3)) \quad (1) \qquad \frac{x^2}{1-x^6} - 2x\tanh^{-1}(x^3)$$

$$(x^2)^2 \quad (1) \qquad x^4$$

$$= \frac{x^2}{x^4(1-x^6)} - \frac{2\tanh^{-1}(x^3)}{x^3} = \frac{1}{x^2(1-x^6)} - \frac{2\tanh^{-1}(x^3)}{x^3}$$

$$= \frac{x}{x^3(1-x^6)} - \frac{2\tanh^{-1}(x^3)(1-x^6)}{x^3(1-x^6)} = \frac{x - 2\tanh^{-1}(x^3)(1-x^6)}{x^3(1-x^6)}$$

If $\tanh x = -\frac{2}{3}$, find $\sinh x$.

SCORE: ____ / 4 PTS

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{-2}{3}$

$\sinh x \cdot \pm\frac{\sqrt{5}}{3} = \frac{-2}{3}$ (*)

~~$\sinh x = \pm\frac{2}{\sqrt{5}}$~~

$\sinh x = \pm\frac{2\sqrt{5}}{5}$ (*)

$1 - \tanh^2 x = \operatorname{sech}^2 x$ (1)

$1 - \left(-\frac{2}{3}\right)^2 = \operatorname{sech}^2 x$

$\frac{5}{9} = \operatorname{sech}^2 x$

$\operatorname{sech} x = \pm\frac{\sqrt{5}}{3}$ (*)

$\cosh x = \pm\frac{3}{\sqrt{5}} = \pm\frac{3\sqrt{5}}{5}$ (*)

Find $\lim_{x\to-\infty} \tanh x$ **algebraically**.

SCORE: 4 / 4 PTS

$$\lim_{x\to-\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}\left(\frac{e^x}{e^x}\right) = \lim_{x\to-\infty} \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{0-1}{0+1} = -1$$

(1/2) (1/2) (1)

For this question, you may use the formulae for $\frac{d}{dx}\sinh x$ and/or $\frac{d}{dx}\cosh x$ without proving them. **SCORE:** 2 **/ 7 PTS**

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the logarithmic formula for $\sinh^{-1} x$**, prove the formula for $\frac{d}{dx}\sinh^{-1} x$.

$\sinh^{-1} x = y$

$x = \sinh y$

$x = \frac{e^y - e^{-y}}{2}\left(\frac{e^y}{e^y}\right)$

$x = \frac{e^{2y} - 1}{2e^y}$

$2e^y x = e^{2y} - 1$

$0 = e^{2y} - 2e^y x - 1$

$e^y = \frac{2x \pm \sqrt{4x^2+4}}{2} = \frac{2x \pm 2\sqrt{x^2+1}}{2} = x \pm \sqrt{x^2+1} = x + \sqrt{x^2+1}$

$y = \ln\left|x + \sqrt{x^2+1}\right|$

$y' = \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{1}{2\sqrt{x^2+1}}(2x)\right)$

$= \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

[b] **Without using the exponential formula for sech x**, prove the formula for $\frac{d}{dx}\operatorname{sech} x$.

$\frac{d}{dx}\operatorname{sech} x = \frac{d}{dx}\frac{1}{\cosh x} = \frac{0(\cosh x) - \sinh x\,(1)\,dx}{(\cosh x)^2}$

$= \frac{-\sinh x}{\cosh^2 x} = \frac{-\sinh x}{\cosh x} \cdot \frac{1}{\cosh x} = -\tanh x \operatorname{sech} x$

(1) (1)

Prove the logarithmic formula for $\tanh^{-1} x$ given in your textbook. **SCORE:** _____ **/ 5 PTS**

NOTE: This is NOT a question about derivatives.

$\tanh^{-1} x = y$

$\tanh y = x$

$\frac{e^y - e^{-y}}{e^y + e^{-y}} = x$

$\frac{e^{2y} - 1}{e^{2y} + 1} = x$ (1)

$e^{2y} - 1 = x(e^{2y} + 1)$

$e^{2y} - 1 - e^{2y}x - x = 0$ (*)

$e^{2y}(1-x) = x + 1$ (1/2)

$e^{2y} = \frac{x-1}{1-x}$

$2y = \ln\left|\frac{x-1}{1-x}\right|$ (*)

$y = \frac{1}{2}\ln\left|\frac{x-1}{1-x}\right|$

(*) (1/2)

[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS: [1] E ✓ [2] D [3] D ✓ [4] D ✓ [5] F ✓ [6] D

[1] Which statement below regarding tests (quizzes, midterms, final exam) is **false** ?

[a] If you continue writing on your test after the stated ending time, you will receive a 0 for that test.
[b] There are no make-ups for missed quizzes.
[c] The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.
[d] If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.
[e] If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.

[2] Which statement below regarding attendance is **false** ?

[a] Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.
[b] Arriving late on a quiz or midterm day will not be counted as late.
[c] Unexcused early departures are considered absences.
[d] If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
[e] Attendance policies will not apply to you if you score more than 80% on every midterm.

[3] How much of your learning does the instructor believe comes from your daily reading and homework combined ?

[a] 40%
[b] 50%
[c] 60%
[d] 70%
[e] 80%

[4] Proper use of the textbook for this class includes

[a] understanding all the terminology used in the book
[b] working out the given examples yourself and checking that you are able to get the same results as the book
[c] reading the sections of the textbook before the corresponding lecture
[d] all of the previous answers [a], [b] and [c]
[e] some, but not all, of the previous answers [a], [b] and [c]

[5] If you score 120 points on Midterm 1, 140 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?
(HINT: You are encouraged to start studying regularly early in the quarter.)

[a] Midterm 1's score will be changed to 145 (the highest midterm score)
[b] Midterm 1's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)
[c] Midterm 1's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 1's and Midterm 2's scores)
[d] Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 1's and the highest midterm's scores)
[e] Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ and Midterm 2's score will be changed to $(140 + 145) \div 2 = 142.5$ (the average of each midterm's and the highest midterm's score)
[f] no midterm scores will be changed

[6] If you score 140 points on Midterm 1, 120 points on Midterm 2 and 145 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

[a] Midterm 2's score will be changed to 145 (the highest midterm score)
[b] Midterm 2's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores)
[c] Midterm 2's score will be changed to $(120 + 140) \div 2 = 132.5$ (the average of Midterm 2's and Midterm 1's scores)
[d] Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 2's and the highest midterm's scores)
[e] Midterm 1's score will be changed to $(140 + 145) \div 2 = 142.5$ and Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of each midterm's and the highest midterm's score)
[f] no midterm scores will be changed